Journal of Engineering Physics and Thermophysics, Vol. 64, No. 5, 1993

THE INITIAL LENGTH OF A CIRCULAR JET OF VARIABLE DENSITY WITH AN ARBITRARY ANGLE OF ITS EJECTION INTO A SWEEPING STREAM

S. K. Voronov and T. A. Girshovich

UDC 532.517.4

An earlier model of flow in an isothermal circular jet developing in a cross stream is extended to the case of a jet of variable density. The initial-length problem is solved.

The ejection of jets at an angle to the main stream is extensively used in practice to intensify heat and mass transfer in different power installations. In [1], a model of flow in an isothermal jet developing in a sweeping stream is proposed and the solution is given to the problem of the initial length of such a jet in which a complex flow is formed by the jet issuing into a sweeping stream. Below, this model is extended to the case of a variable-density jet developing in a sweeping stream, and the integral-method solution is given to the problem of the initial length of such a jet at an arbitrary angle of its ejection into the main stream.

When a jet is ejected into a sweeping stream of another density, we may assume that the deformation rate of the jet boundaries v_a, just as in the case of an incompressible fluid [1], is proportional to the sweeping-stream velocity component normal to the jet trajectory. But, since a less dense jet must be more permeable for a denser cross stream, and vice versa, and the basic parameter, which determines the flow in a jet in a sweeping stream, is the jet and stream velocity head ratio, it is logical to assume that v_a is also proportional to the square root of the stream and jet gas density ratio ρ_∞/ρ_0, i.e.,

$$v_a = \varepsilon V_{\infty n}\left(\frac{\rho_\infty}{\rho_0}\right)^{1/2}. \tag{1}$$

The rates of expansion of the jet boundaries in the plane of symmetry and in the lateral direction will be written in the same forms as in [1]:

$$y'_{1m} = r'_\delta - \frac{v_a}{u_0},\quad z'_{1m} = r'_\delta + \frac{v_a}{u_0}, \tag{2}$$

but instead of the rate of expansion of an incompressible fluid jet these formulas should involve the rate of growth of the boundaries of a variable-density submerged jet for a jet ejected at an angle of 90° to the stream or of a variable-density jet in a concurrent stream for a jet ejected into the main stream at an angle other than 90°.

It is clear that the relation between the rates of jet expansions in the plane of symmetry y'_{1m} and in the lateral direction z'_{1m} remains the same as in the case of an incompressible fluid jet:

$$z'_{1m} = y'_{1m} + 2\frac{v_a}{u_0}. \tag{3}$$

In conformity with the above, the factor $(\rho_\infty/\rho_0)^{1/2}$ should be introduced into the additional discharge of the fluid ejected by a jet developing in a sweeping stream as compared with the discharge of the fluid ejected by an ordinary jet. As a result, the value of the additional relative discharge of the fluid $q_{add} = G_{add}/(2\pi r_0^2\rho_0 u_0)$ will be equal to

$$q_{add} = \frac{\varepsilon}{2\pi}\left(\frac{\rho_\infty}{\rho_0}\right)^{1/2}\frac{V_{\infty n}}{u_0}\int\limits_0^x z_{1m}dx. \tag{4}$$

S. Ordzhonikidze Moscow Aviation Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 64, No. 5, pp. 529-536, May, 1993. Original article submitted April 22, 1992.

1062-0175/93/6405-0427$12.50

It should be noted that the above generalization of the model of flow in an incompressible-fluid circular jet developing in a cross stream can also be used for the case of a two-phase jet in a cross stream and for the case of a purely gas jet in a gas cross stream with a solid or liquid impurity. In this case, as the value r_δ' in Eqs. (2) we should use the rate of growth of the submerged two-phase jet boundary (when the angle of ejection is $\alpha_0 = 90^0$) or of the two-phase jet in a concurrent stream. The gas and particle mixture density in a jet should be determined from the equation

$$\rho_0 = \rho_g (1 + \gamma_0). \tag{5}$$

Suppose that a circular jet is ejected at a certain angle to the stream of another gas. In the initial length of such a jet there are the core of constant total pressure and a mixing zone with different rates of expansion in the plane of symmetry and in the lateral direction. We will assume that:

1) the jet temperature axis in the initial length coincides with the dynamic axis;

2) the velocity profiles in the cross sections of the jet are similar and can be described by the Schlichting formula;

3) the temperature (enthalpy) profiles in the cross sections of the jet are similar and differ from the velocity profiles;

4) the jet axis is the stream line;

5) the lines of equal velocities are ellipses;

6) the jet velocity and temperature boundaries coincide.

If velocity and temperature profiles (impurity concentrations) are described by universal relations assuming the ellipticity of the jet cross section, then to describe the flow pattern in the initial length of such a jet, one should know four values: the boundaries of the jet and the boundaries of the constant total pressure core in the plane of symmetry and in the lateral direction. Just as in the case of an incompressible fluid jet, in order to determine the above values, one can use: the condition of conservation of the jet excess momentum, the integral discharge equation involving the additional discharge ejected by the jet and determined with the aid of Eq. (4), an equation which connects the rates of jet expansion in the plane of symmetry and in the lateral direction (Eq. (3)), and an equation which automatically results from the condition that the lines of equal velocities are ellipses, i.e.,

$$4 \int_0^{\pi/2} d\varphi \int_0^{r_1(\varphi)} \rho u (u - u_\delta)\, r dr = \text{const}, \tag{6}$$

$$4 \int_0^{\pi/2} d\varphi \int_0^{r_1(\varphi)} \rho u r dr = G_0 + G_э + G_{\text{add}}, \tag{7}$$

$$z_{1m}' = y_{1m}' + 2\varepsilon \left(\frac{\rho_\infty}{\rho_0}\right)^{1/2} \frac{V_{\infty n}}{u_0}, \quad \frac{y_{2m}}{y_{1m}} = \frac{z_{2m}}{z_{1m}}. \tag{8}$$

The scheme of the jet is shown in Fig. 1.

As noted above, the velocity and temperature profiles entering into Eqs. (6) and (7) may be considered similar and can be described with the aid of the equations

$$\frac{u - u_\delta}{u_0 - u_\delta} = 2[\eta(\varphi)]^{3/2} - [\eta(\varphi)]^3, \quad \frac{T - T_\delta}{T_0 - T_\delta} = \eta(\varphi), \quad \eta(\varphi) = \frac{r_1(\varphi) - r}{r_1(\varphi) - r_2(\varphi)},$$

$$r_1(\varphi) - r_2(\varphi) = r_\delta(\varphi), \tag{9}$$

whereas the density is determined from the equation of state, which yields

$$\rho/\rho_0 = T_0/T. \tag{10}$$

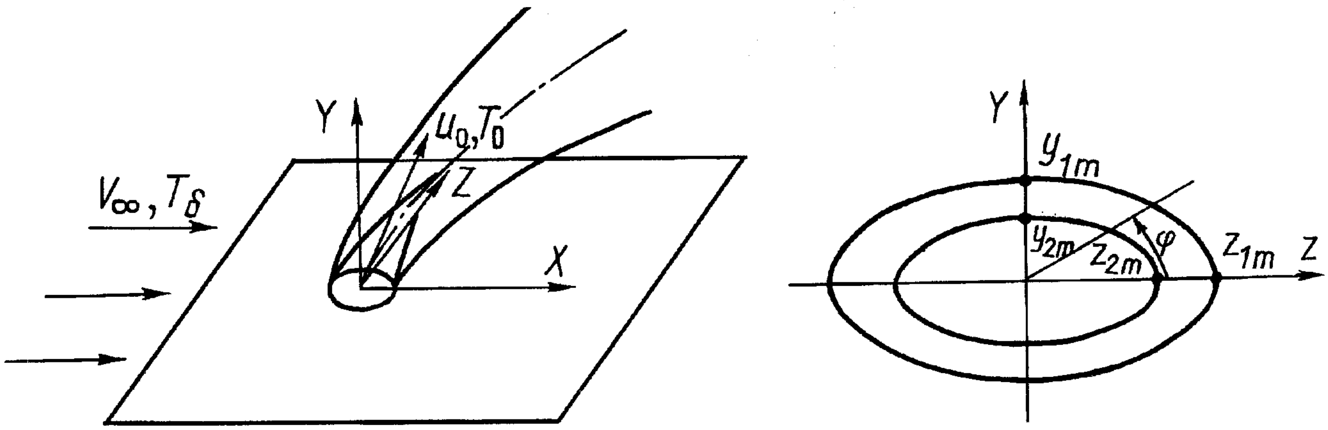

Fig. 1. Scheme of the jet.

The discharge of the fluid ejected by a circular nonisothermal jet in a concurrent stream compounded with the initial jet discharge can be determined from the equation

$$q_1 = \frac{G_0 + G_э}{2\pi r_0^2 u_0 \rho_0} = \int_0^{y_1} \frac{\rho u}{\rho_0 u_0} y\,dy = B_2 y_1^2 + B_6 y_1 y_2 + B_4 y_2^2, \tag{11}$$

$$B_2 = A_{13} - A_{14} + m_c (A_{01} - A_{13} - A_{02} + A_{14}),\ B_4 = 0{,}5 - A_{14} - m_c (A_{02} - A_{14}),$$
$$B_6 = 2A_{14} - A_{13} + m_c (A_{13} + 2A_{02} - A_{01} - 2A_{14}).$$

Here

$$m_c = \frac{u_\delta}{u_0};\ u_\delta = V_\infty \cos\alpha,\ \frac{u - u_\delta}{u_0 - u_\delta} = 2\eta^{3/2} - \eta^3 = f(\eta),\quad \eta = \frac{y_1 - y}{y_1 - y_2}, \tag{12}$$

$$A_{01} = \int_0^1 \frac{\rho}{\rho_0} d\eta,\ A_{02} = \int_0^1 \frac{\rho}{\rho_0} \eta\,d\eta,$$
$$A_{13} = \int_0^1 \frac{\rho}{\rho_0} f(\eta)\,d\eta,\ A_{14} = \int_0^1 \frac{\rho}{\rho_0} f(\eta)\,\eta\,d\eta. \tag{13}$$

The ordinates of the jet mixing zone boundaries in a concurrent stream y_1 and y_2 can be determined by solving the problem of the initial length of a circular nonisothermal jet in a concurrent stream. However, on the assumption that the mixing zone boundaries are linear, which is confirmed by experiments, y_1 and y_2 can be determined from the formulas

$$y_1 = 1 + k_1\delta,\ y_2 = 1 - k_2\delta,\ k_1 + k_2 = 1. \tag{14}$$

Since $y_1 = \delta = \delta_{ij}$ at the end of the initial length (δ_{ij} is the half-width of the jet at the end of the jet initial length in a concurrent stream), we have

$$k_1 = (\delta_{ij} - 1)/\delta_{ij},\ k_2 = 1/\delta_{ij}. \tag{15}$$

The magnitude of the jet boundary ordinate at the end of the initial length can easily be obtained from the equation of excess momentum conservation in the jet at $y_2 = 0$, whence we have

$$\delta_{ij} = (2B_1)^{-1/2},\ B_1 = A_{21} - A_{12} + m_c (A_{13} - A_{14} - A_{21} + A_{22}). \tag{16}$$

The quantity $q_1(x)$ can be determined from the equation of discharge for a jet in a concurrent stream on the assumption that the change in the discharge is proportional to the excess fluid velocity in the jet and to the ordinate of the jet boundary

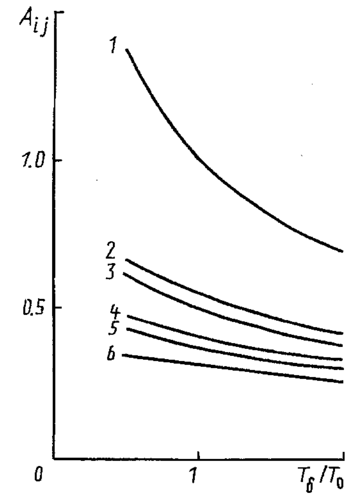

Fig. 2. Integrals as functions of $\Theta = T_\delta/T_0$: 1) A_{01}; 2) A_{13}; 3) A_{02}; 4) A_{21}; 5) A_{14}; 6) A_{22}.

$$\frac{d}{dx}\int_0^{y_1} \rho u y dy = c_1 \rho_\infty (u_0 - u_\delta) y_1, \tag{17}$$

whence, after simple manipulation, we obtain

$$q_1(x) = 0{,}5 + c_1 x (1 - m) \frac{\rho_\infty}{\rho_0}\left(1 + \frac{\delta_{\text{нс}} - 1}{2} \frac{x}{x_{\text{нс}}}\right), \tag{18}$$

where x_{ij} is the initial length of a nonisothermal jet in a concurrent stream.

The quantity x_{ij} can be determined from Eq. (11) at $y_2 = 0$ and at the value of $q_1(x)$ found from Eq. (18):

$$x_{\text{ij}} = \frac{2\delta_{\text{ij}}^2 B_2 - 1}{c_1 \frac{\rho_\infty}{\rho_0}(1 - m)(1 + \delta_{\text{ij}})}. \tag{19}$$

Here c_1 is an empirical constant which can be obtained from the relation for the width of the submerged isothermal circular jet mixing zone $\delta = cx$, from which we obtain that at the end of the initial length $c = \delta_i/x_i$. By calculating the integrals A_{13}, A_{14}, A_{21}, and A_{22} for an isothermal jet and performing simple computations, we obtain

$$c_1 = 0{,}1c, \tag{20}$$

where c, as is known from experiments, is approximely equal to 0.25-0.3.

The values A_{01}, A_{02}, A_{13}, A_{14}, A_{21}, and A_{22} in Eqs. (11) and (16) are integrals defined with the aid of Eq. (13)

$$A_{21} = \int_0^1 \frac{\rho}{\rho_0} f^2(\eta)\, d\eta, \quad A_{22} = \int_0^1 \frac{\rho}{\rho_0} f^2(\eta)\, \eta d\eta. \tag{21}$$

Figure 2 shows integrals (13) and (21) as functions of the ratio T_δ/T_0. They are calculated on the assumption that for a substantially subsonic nonisothermal jet the temperature profiles are similar and are described by the equation

$$\frac{T - T_\delta}{T_0 - T_\delta} = \eta, \quad \eta = \frac{y_1 - y}{y_1 - y_2}. \tag{22}$$

The value of the additional discharge of the fluid ejected by the jet developing in a sweeping stream will be found from Eq. (4), in which z_{1m} can be obtained in a first approximation with the aid of Eq. (3) and the first of relations (2) and (14)-(16). Simple manipulations yield

$$z_{1m} = 1 + \left[\frac{\delta_{ij} - 1}{x_{ij}} + \varepsilon \left(\frac{\rho_\infty}{\rho_0} \right)^{1/2} \frac{V_{\infty n}}{u_0} \right] x \tag{23}$$

and

$$q_{add} = \frac{\varepsilon}{2\pi} \left(\frac{\rho_\infty}{\rho_0} \right)^{1/2} \frac{V_{\infty n}}{u_0} \left\{ x + \left[\frac{\delta_{ij} - 1}{x_{ij}} + \varepsilon \left(\frac{\rho_\infty}{\rho_0} \right)^{1/2} \frac{V_{\infty n}}{u_0} \right] \frac{x^2}{2} \right\}. \tag{24}$$

Then, since $q(x) = q_1(x) + q_{add}(x)$, we obtain the relation for $q(x)$ in view of Eqs. (18) and (24)

$$q(x) = 0{,}5 + b_2 x + a_2 x^2, \tag{25}$$

where

$$a_2 = c_1 (1 - m) \frac{\rho_\infty}{\rho_0} \frac{(\delta_{ij} - 1)}{2x_{ij}} + \frac{\varepsilon}{4\pi} \left(\frac{\rho_\infty}{\rho_0} \right)^{1/2} \frac{V_{\infty n}}{u_0} \left[\frac{\delta_{ij} - 1}{x_{ij}} + \right.$$
$$\left. + \varepsilon \left(\frac{\rho_\infty}{\rho_0} \right)^{1/2} \frac{V_{\infty n}}{u_0} \right], \quad b_2 = c_1 (1 - m) \frac{\rho_\infty}{\rho_0} + \frac{\varepsilon}{2\pi} \left(\frac{\rho_\infty}{\rho_0} \right)^{1/2} \frac{V_{\infty n}}{u_0}. \tag{26}$$

Taking into account the similarity of the velocity and temperature profiles (9) and substituting Eqs. (9) and (25) into (6) and (7), we find

$$\begin{gathered} y_{1m} z_{1m} B_1 + y_{1m} z_{2m} B_5 + y_{2m} z_{2m} B_3 = 0{,}5, \\ y_{1m} z_{1m} B_2 + y_{1m} z_{2m} B_6 + y_{2m} z_{2m} B_4 = q(x), \end{gathered} \tag{27}$$

where B_2, B_4, and B_6 are determined from Eqs. (11), and B_3 and B_5 from the equations

$$\begin{gathered} B_3 = 0{,}5 - A_{22} + m_c (A_{22} - A_{14}), \\ B_5 = 2A_{22} - A_{21} + m_c (2A_{14} - A_{13} + A_{21} - 2A_{22}). \end{gathered} \tag{28}$$

The left-hand sides of Eqs. (27) are calculated with the use of easily computed integrals

$$\int_0^{\pi/2} r_1^2 d\varphi = \frac{\pi}{2} y_{1m} z_{1m}, \quad \int_0^{\pi/2} r_1 r_2 d\varphi = \frac{\pi}{2} y_{1m} z_{1m}, \quad \int_0^{\pi/2} r_2^2 d\varphi = \frac{\pi}{2} y_{2m} z_{2m}. \tag{29}$$

Multiplying both sides of the first equation of system (27) by $q(x)$ and both sides of the second equation of system (27) by 0.5 and subtracting the second equation from the first, after simple manipulations we find

$$a_1 f^2 + b_1 f + c_1 = 0. \tag{30}$$

Here

$$f = \frac{y_{2m}}{y_{1m}} = \frac{z_{2m}}{z_{1m}}, \tag{31}$$

$$a_1 = B_3 q(x) - 0{,}5 B_4, \quad b_1 = B_5 q(x) - 0{,}5 B_6, \quad c_1 = B_1 q(x) - 0{,}5 B_2. \tag{32}$$

Solving Eq. (30) with allowance for the fact that $0 \le f \le 1$, we have

$$f = \frac{y_{2m}}{y_{1m}} = \frac{(b_1^2 - 4a_1 c_1)^{1/2} - b_1}{2a_1}, \tag{33}$$

$$y_{2m} = y_{1m} f, \quad z_{2m} = z_{1m} f. \tag{34}$$

From the first of Eqs. (27), using Eq. (34), we find

$$y_{1m} z_{1m} = n, \quad n = \frac{0{,}5}{B_3 f^2 + B_5 f + B_1}. \tag{35}$$

The value of z_{1m} is determined with the help of Eq. (8) and turns out to be equal to

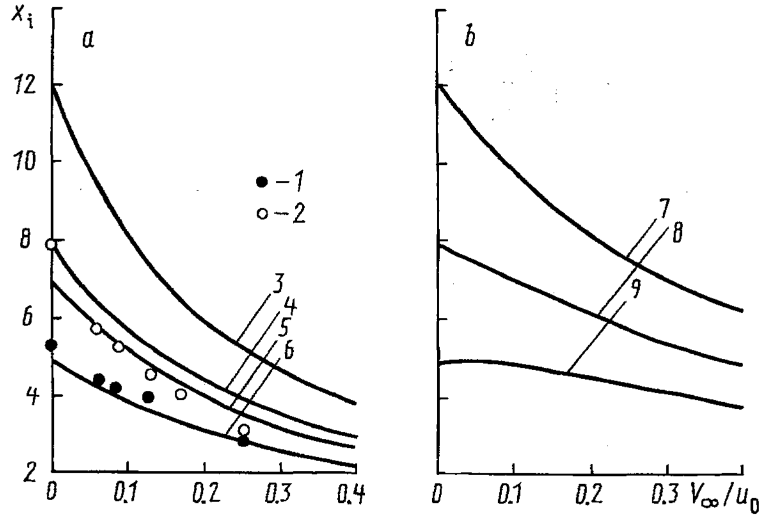

Fig. 3. Initial length as a function of the stream and jet velocity ratio $m = V_\infty/u_0$: a) $\alpha = 90°$, experiment [4]: 1) $\Theta = 0.5$; 2) 0.877; calculation: 3) $\Theta = 2.0$; 4) 1.0; 5) 0.877; 6) 0.5; b) $\alpha = 60°$, calculation: 7) $\Theta = 2.0$; 8) 1.0; 9) 0.5.

$$z_{1m} = y_{1m} + 2\varepsilon\left(\frac{\rho_\infty}{\rho_0}\right)^{1/2}\frac{V_{\infty n}}{u_0}x. \tag{36}$$

Substituting Eq. (36) into (35) and solving the resulting equation for y_{1m}, we find

$$y_{1m} = \left\{\left[\varepsilon\left(\frac{\rho_\infty}{\rho_0}\right)^{1/2}\frac{V_{\infty n}}{u_0}x\right]^2 + n\right\}^{1/2} - \varepsilon\left(\frac{\rho_\infty}{\rho_0}\right)^{1/2}\frac{V_{\infty n}}{u_0}x. \tag{37}$$

The initial length for a jet in a sweeping stream x_i can be found with the aid of Eq. (27), having assumed that $x = x_i$ and $y_{2m} = z_{2m} = 0$ and solving these equations for x_i, whence

$$x_i = \frac{(b_2^2 + 4a_2c_2)^{1/2} - b_2}{2a_2}, \tag{38}$$

where

$$c_2 = 0{,}5\left(\frac{B_2}{B_1} - 1\right). \tag{39}$$

The characteristics of the initial length of a nonisothermal circular jet in a sweeping stream can be calculated in the following order:

1) Using the given temperatures and velocities of the stream and the jet, determine the temperature ratio $\Theta = T_\delta/T_0$ and the velocity ratio $m = V_\infty/u_0$ in the stream and the jet.

2) For the given ejection angle α_0, determine from the following formulas the concurrent stream velocity and the sweeping stream velocity normal to the jet trajectory, as well as the concurrency parameter:

$$u_\delta = V_\infty \cos\alpha_0, \quad V_{\infty n} = V_\infty \sin\alpha_0, \quad m_c = u_\delta/u_0. \tag{40}$$

3) Using Fig. 2 and the value of Θ found under item 1, determine the values of the integrals A_{13}, A_{14}, A_{21}, A_{22}, A_{01}, and A_{02}.

4) From Eqs. (16) and (19), using the values of m_c and integrals obtained under items 2 and 3, determine the initial length of the jet in a concurrent stream x_y and the jet radius at the end of the initial length δ_{ij}.

5) Calculate the quantities a_2 and b_2 from Eqs. (26) and c_2 from Eq. (39).

6) From Eq. (38), determine the initial length of the nonisothermal jet in a sweeping stream.

7) Calculate the values of y_{1m}, z_{1m}, y_{2m}, and z_{2m} at different distances to the nozzle outlet x in the following

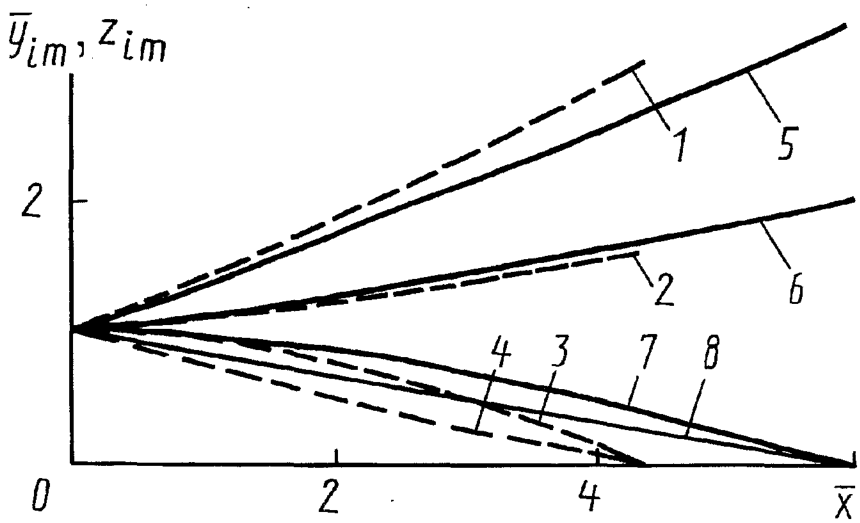

Fig. 4. The calculated boundaries of the mixing zone at $\alpha = 90^o$, $V_\infty/u_0 = 0.2$, $\Theta = 1.0$: 1) z_{1m}; 2) y_{1m}; 3) z_{2m}; 4) y_{2m}; $\Theta = 2.0$: 5) z_{1m}; 6) y_{1m}; 7) z_{2m}; 8) y_{2m}.

order:

a) from the given x values, the values of q(x) are calculated from Eq. (25), and a_1, b_1, c_1 from Eq. (32);

b) the values of $f = y_{2m}/y_{1m}$ are determined from Eq. (33), and the values of n from Eq. (35);

c) the values of y_{1m} are obtained from Eq. (37), and of z_{1m} from Eq. (26), as well as the values of y_{2m}, z_{2m} from Eq. (34).

It must be emphasized that increasing the jet ejection angle above $\alpha_0 = 90^o$ should result in jet development in the countercurrent stream near the nozzle outlet (in the initial length). But in this case, as is known (see [2, 3]), the boundary layer thickening angle is independent of the velocity ratio and has the same value as for a submerged jet. Therefore, in formulas for calculating the initial-length characteristics it is necessary to set $m_c = 0$. It is evident that in such a case the jet ejection angle should influence the jet characteristics only through the change in the sweeping stream velocity component normal to the jet trajectory. Since the sweeping velocity decreases with an increase in the jet ejection, the additional ejection into the jet also decreases. Consequently, the initial length should also increase.

Figure 3 shows the initial length as a function of the stream-to-jet ratio $m = V_\infty/u_0$ at $\alpha_0 = 90^o$. The plots with the results of calculation at $\Theta \simeq 1.0$ and $\Theta \simeq 0.5$ display the experimental data of Hendrixon [4]. It is seen that the predicted results agree satisfactorily with experimental data.

Analysis of the predicted data at different jet ejection angles shows that, as expected, an increase of the value of V_∞/u_0 leads to a decrease in the initial length when $\alpha_0 \geq 60^o$. With the jet ejection angle $\alpha_0 < 60^o$, the concurrent stream velocity (the sweeping stream velocity component directed along the jet trajectory) is so great that its influence is dominant, and the initial length increases with V_∞/u_0. Thereafter the rate of increase diminishes and when $V_\infty/u_0 >$ 0.2-0.3, a further rise in V_∞/u_0 leads to shortening of the initial length.

Figure 4 shows the influence of the flow nonisothermicity on the development of the jet mixing zone boundaries in the plane of symmetry and in the lateral direction in case of cold jet ejection into a hotstream (T_δ/T_0 = 2.0) at $V_\infty/u_0 = 0.2$ and $\alpha_0 = 90^o$. It is clear that in this case the flow nonisothermicity leads to an increase in the jet range and a decrease in the mixing zone width. Note that in view of the absence of experimental data, it has not been possible as yet to verify the results of calculation of the mixing zone boundaries. However, the qualitative results of calculation agree with available concepts about the development of the nonisothermal jet initial length in a sweeping stream.

Thus, the proposed solution of the problem makes it possible with the aid of simple formulas to describe a complex flow in the initial length of a nonisothermal jet in a sweeping stream, considering the specific features of the development of such a jet.

NOTATION

c, empirical constant (the degree of the isothermal submerged jet mixing zone expansion); G_c, initial fluid mass discharge in a jet; G_{add}, additional mass discharge of fluid ejected by a jet in a cross stream; G_e, mass discharge of fluid ejected by an ordinary jet (submerged or in a concurrent stream); G_g, G_{im}, gas and impurity mass discharges in the case of a two-phase jet; r, φ, x, cylindrical coordinates along the jet trajectory (see Fig. 1); T, temperature; u, longitudinal velocity component; v_a, rate of deformation of jet boundaries; V_∞, velocity of sweeping stream; $V_{\infty n}$, sweeping stream velocity component normal to the jet trajectory; y_{1m}, z_{1m}, ordinates of jet boundaries in the plane of symmetry and in the lateral direction, respectively; y_{2m}, z_{2m}, ordinates of core boundaries in the plane of symmetry and in the lateral direction; α_0, angle of jet ejection (angle between the jet and stream directions); $\gamma_0 = G_{im}/G_g$, impurity concentration; ε, empirical constant; ρ_0, ρ_∞, gas densities of jet and sweeping stream; ρ_g, density of gas phase. Subscripts: 0, initial; δ, on jet boundary.

REFERENCES

1. T. A. Girshovich, Inzh.-Fiz. Zh., **54**, No. 6, 905-912 (1988).
2. G. N. Abramovich, Theory of Turbulent Jets [in Russian], Moscow (1960).
3. G. N. Abramovich, T. A. Girshovich, S. Yu. Krasheninnikov, et al., Theory of Turbulent Jets [in Russian], Moscow (1984).
4. V. A. Hendrixon, Turbulent Jet Flows [in Russian], Tallinn (1982), pp. 65-73.